Quasistatically controlled bianisotropic media: Dual composite materials

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A material concept of the electrostatically controlled bianisotropic materials (ECBMs) is introduced. Composite materials are based on electrically small piezoelectric resonators with an aperture in a perfectly conducting surface. The ECBMs are dual composite materials with respect to the magnetostatically controlled bianisotropic materials conceptualized recently by the present author. [S1063-651X(98)11512-X]

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I. INTRODUCTION

The electromagnetics of bianisotropic materials holds the key to many important technologies. In microwaves, these bianisotropic materials are composite materials. The main feature of the known bianisotropic composites (based on helices or Ω particles) is the first-order role played by the size parameters ka in the emergence of the magnetoelectric properties; here a is the particle size and k is the wave number in the host material. For this reason, the electric and magnetic fields are not curl-free away from the particle and the quasistatic effective-medium theories may be applicable only for dilute composites. In other words, such media are modeled as a gas of scatterers. Much needs to be done, however, before these bianisotropic composites come to be used in microwave applications [1–3].

A different class of bianisotropic materials has been conceptualized recently by the present author. These are particulate composites based on small ferromagnetic resonators with a special-form surface metallization [4,5]. The main point is that the dyadic polarizabilities of every bianisotropic particle are obtained by solving magnetostatic problems and the particle may be considered as a glued pair of two (electric and magnetic) dipoles. Since a bianisotropic particle is described quasistatically and the electric and magnetic fields are curl-free at every point away from the particle, the effective-medium theories for dense homogenized materials may be successfully used [6]. In this case, we have ''solid state matter'' in comparison to ''gas matter'' based on a composition of helices or Ω particles. A vast number of fundamental problems and applications (waveguide and resonator structures, antenna substrates, etc.) are emerging from theoretical and experimental works based on these composite materials [4,5].

Lakhtakia suggested to name media described initially in [4] as *magnetostatically controlled bianisotropic materials* (MCBMs) [7]. Now the question arises, can one conceive of dual composite materials that may be named *electrostatically controlled bianisotropic materials* (ECBMs)? In this paper we will show that ECBMs may be realized based on elastodynamic quasielectrostatic oscillations in piezoelectric resonators with an aperture in a perfectly conducting surface. When the ECBMs are conceptualized together with the MCBMs, we can assert that a general class of dual quasistatically (magnetostatically and electrostatically) controlled bianisotropic materials exists.

II. CURL-FREE-FIELD BIANISOTROPIC COMPOSITES: A CONCEPT OF DUALITY

In a general case, the electromagnetic wave processes in media may be accompanied by vortex $\mathbf{E}_c,\mathbf{H}_c$ and potential $\mathbf{E}_p,\mathbf{H}_p$ electric and magnetic fields, so that the total fields are represented as

$$\mathbf{E}=\mathbf{E}_c+\mathbf{E}_p, \quad \mathbf{H}=\mathbf{H}_c+\mathbf{H}_p, \tag{1}$$

where

$$\mathbf{E}_p=-\nabla\phi, \quad \mathbf{H}_p=-\nabla\psi, \\ \nabla\cdot\mathbf{E}_c=0, \quad \nabla\cdot\mathbf{H}_c=0. \tag{2}$$

The scalar electric and magnetic potentials ϕ and ψ are caused by electric and magnetic charges, respectively, in accordance with the Poisson equation (the Coulomb gauge) [8].

The quasistatic (potential) wave propagation may take place in media. These potential waves are due to short-length interactions between adjacent polarization vectors (magnetostatic waves in ferromagnetics, elastodynamic quasielectrostatic waves in piezoelectrics) or due to Coulomb interaction between the mobile charges (space-charge waves). Barybin characterized such media with potential-wave propagation as *active polarized media* [9]. In the classical description, one of the main reasons why a medium can support the propagation of potential waves arises from the kinematics of particles. There are, in particular, ac magnetization motion in ferromagnetics [10,11] or the particle displacement in piezoelectrics [12,13]. Due to the mechanical processes, we have additional (in comparison to the energy balance in ''pure'' electromagnetic waves) mechanisms of storage and exchange of energy. In nature, we have quasimagnetostatic ($\nabla\times\mathbf{H}\simeq0$) or quasielectrostatic ($\nabla\times\mathbf{E}\simeq0$) waves. Thus the potential-wave propagation in active polarized media is accompanied by curl fields. There are curl electric field in quasimagnetostatic waves in ferromagnetics and curl magnetic field in quasielectrostatic waves in piezoelectrics. The quasistatic-wave propagation may also be accompanied by surface currents: electric surface current in quasimagnetostatic waves and magnetic surface current in quasielectrostatic waves. Obviously, bulk currents cannot take place since a quasistatic description inside the current region is impossible. In our further consideration we will use the following terms: magnetostatic waves (MSWs) for potential

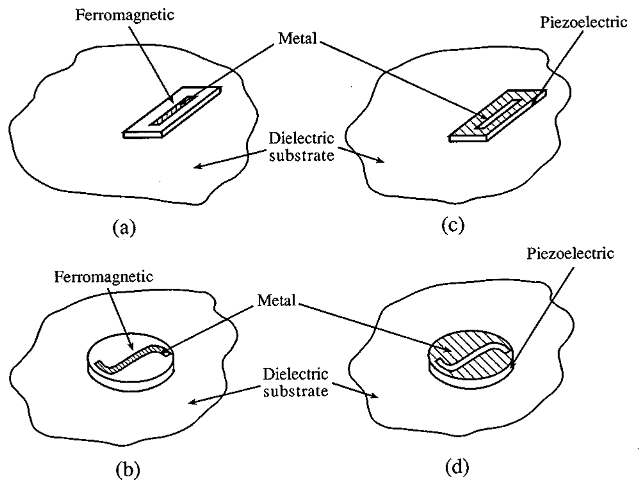

FIG. 1. Dual bianisotropic particles.

waves in ferromagnetics and electrostatic waves (ESWs) for potential waves in piezoelectrics.

Let us consider a small ferromagnetic resonator with a special-form region of surface metallization. When quasistatic oscillations due to the MSW process take place, the resonator may have properties of a bianisotropic particle with the curl-free fields outside the particle. Every bianisotropic particle is a glued pair of two (magnetic and electric) dipoles: The magnetic dipole is due to the ferrite body and the electric dipole is due to the metalization region. Bianisotropic materials composed of MSW ferromagnetic resonators were described and analyzed in [4,5]. The theoretical analysis was based on the theory of excitation of MSW waveguides [14].

Now let us consider a small piezoelectric resonator with an aperture in a perfectly conducting surface. We can assert *a priori* that when quasistatic oscillations due to the ESW process take place, the resonator may have properties of bianisotropic particle with a glued pair of two dipoles. In this case, the electric dipole is due to the piezoelectric body and the magnetic dipole is due to an aperture in a metallic screen.

Our conceptual analysis enables us to represent two types of dual quasistatic (magnetostatic and electrostatic) bianisotropic particles. In Fig. 1 we can see magnetostatic bianisotropic particles based on ferromagnetic resonators with surface metallic strips [Figs. 1(a) and 1(b)] [4,5] in comparison with electrostatic bianisotropic particles based on piezoelectric resonators with apertures in metallic screens [Figs. 1(c) and 1(d)]. One can compose two types of dual curl-free-field bianisotropic materials: the MCBMs and the ECBMs. Thus a careful analysis has to be made to show that electrostatically controlled bianisotropic particles may be realized.

III. ELECTROSTATICALLY CONTROLLED BIANISOTROPIC PARTICLES

A piezoelectric resonator with an aperture in a perfectly conducting surface is considered as a bianisotropic particle. Because of quasielectrostatic (short-wavelength) oscillations in a piezoelectric resonator, we can distinguish the intrinsic quasielectrostatic mode fields and the external (given) fields. Induced electric $\mathbf{p}$ and magnetic $\mathbf{m}$ dipole moments are related to the external electric $\mathbf{E}^{\rm ext}$ and magnetic $\mathbf{H}^{\rm ext}$ fields as

$$\mathbf{p}=\boldsymbol{\alpha}_{ee}\cdot\mathbf{E}^{\rm ext}+\boldsymbol{\alpha}_{em}\cdot\mathbf{H}^{\rm ext},\quad \mathbf{m}=\boldsymbol{\alpha}_{me}\cdot\mathbf{E}^{\rm ext}+\boldsymbol{\alpha}_{mm}\cdot\mathbf{H}^{\rm ext}. \tag{3}$$

To obtain the dyadic polarizabilities $\boldsymbol{\alpha}_{ee}$, $\boldsymbol{\alpha}_{em}$, $\boldsymbol{\alpha}_{me}$, and $\boldsymbol{\alpha}_{mm}$ one has to solve a problem of excitation of a piezoelectric resonator by the external (given) fields.

Piezoelectric resonators with apertures are characterized as structures with complex geometries. One of the ways to solve the excitation problem is a decomposition of a resonator by waveguide sections and an analysis of excitation in these waveguide sections. In this paper we consider a piezoelectric resonator depicted in Fig. 1(c) as a composition of piezoelectric-waveguide sections. A similar decomposition was used in an analysis of bianisotropic particles in MCBMs [4]. So the problem is reduced to an analysis of mode excitation in piezoelectric waveguides by the external electric and magnetic fields. When the problem of the piezoelectric waveguide excitation by the external fields is solved, further generalization for piezoelectric resonators may be realized.

IV. MODE EXCITATION IN PIEZOELECTRIC WAVEGUIDES BY THE EXTERNAL FIELDS

In different types of waveguides described in the literature (electromagnetic [15,16], MSW [10,11], and acoustic [12,13]), normal mode excitation is analyzed as excitation by the external (given) electric and magnetic currents and charges. For some waveguide problems, another type of excitation may be considered however. There is mode excitation due to the external (given) electric and magnetic fields. This kind of excitation problem is especially important in our case when induced dipole moments of a bianisotropic particle are defined. Quasistatically controlled oscillating processes in bianisotropic particles of the MCBMs or ECBMs have scales of space variations much less than the corresponding scales of the external electric and magnetic fields. This makes it possible to distinguish the intrinsic quasistatic mode fields and the external (given) electric and magnetic fields.

To solve an excitation problem in a waveguide, one can use a complete functional basis of eigenmodes of a nonexcited waveguide. To obtain this functional basis, a correct eigenvalue problem has to be set up. Let steady-state time-harmonic ($e^{i\omega t}$) field variations along the longitudinal z axis of a multilayered piezoelectric waveguide be described by the factor $e^{-\gamma z}$. Based on the electro-magnetic and acoustic field equations and piezoelectric constitutive relations [12,13], one can write the eigenvalue equation for every layer j of a piezoelectric waveguide. In the quasielectrostatic approximation ($\mathbf{E}=-\boldsymbol{\nabla}\phi$), two forms of the eigenvalue equation in a piezoelectric waveguide are possible. One form may be written as

$$([G_\perp]^{(j)}-\gamma[R_1])[V]^{(j)}=0, \tag{4}$$

where $[G_\perp]$ is the differential-matrix operator and $[R_1]$ is the matrix coefficient,

$$[G_\perp]=\begin{pmatrix} -i\omega\rho & \overline{\boldsymbol{\nabla}}_{\cdot}^{(\perp)} & 0 & 0 \\ \overline{\boldsymbol{\nabla}}_s^{(\perp)} & -i\omega\mathbf{S}^E: & i\omega\mathbf{d}\cdot\boldsymbol{\nabla} & 0 \\ 0 & 0 & 0 & -i\omega\boldsymbol{\nabla}_\perp\cdot \\ 0 & i\omega(\boldsymbol{\epsilon}^T)^{-1}(\mathbf{d}:) & i\omega\boldsymbol{\nabla}_\perp & i\omega(\boldsymbol{\epsilon}^T)^{-1} \end{pmatrix}, \tag{5}$$

$$[R_1]=\begin{pmatrix} 0 & \overline{\boldsymbol{\nabla}}_{\cdot}^{(\parallel)} & 0 & 0 \\ \overline{\boldsymbol{\nabla}}_s^{(\parallel)} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\omega\mathbf{e}_z\cdot \\ 0 & 0 & i\omega\mathbf{e}_z\cdot & 0 \end{pmatrix}, \tag{6}$$

and $[V]$ is the vector function

$$[V]=\begin{pmatrix} \mathbf{v} \\ \mathbf{T} \\ \phi \\ \mathbf{D} \end{pmatrix}. \tag{7}$$

Here $\mathbf{e}_z$ is the unit vector along the axis z and $\boldsymbol{\nabla}_\perp$ and $\boldsymbol{\nabla}_\perp\cdot$ are the transverse gradient and divergence, respectively. In Eqs. (5)–(7) we used designations from Auld's book [12]: $\mathbf{S}$ is the strain field, $\mathbf{T}$ is the stress field, $\mathbf{v}$ is the particle velocity, and $\mathbf{d}$ is the piezoelectric strain constant. Other designations can be found in [12]. Transverse and longitudinal parts of the divergence of stress $\overline{\boldsymbol{\nabla}}^{(\perp)}\cdot\mathbf{T}$ and $\overline{\boldsymbol{\nabla}}^{(\parallel)}\cdot\mathbf{T}$ and transverse and longitudinal parts of the symmetric gradient of the particle velocity $\overline{\boldsymbol{\nabla}}_s^{(\perp)}\mathbf{v}$ and $\overline{\boldsymbol{\nabla}}_s^{(\parallel)}\mathbf{v}$ are defined, respectively, by the matrix coefficients (in Auld's notation [12])

$$\overline{\boldsymbol{\nabla}}_{\cdot}^{(\perp)}=\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix},$$

$$\overline{\boldsymbol{\nabla}}_{\cdot}^{(\parallel)}=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

$$\overline{\boldsymbol{\nabla}}_s^{(\perp)}=\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \\ 0 & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}, \quad \overline{\boldsymbol{\nabla}}_s^{(\parallel)}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{9}$$

Together with the form (4), another form of the eigenvalue equation is possible

$$([M_\perp]^{(j)}-\gamma[R_2])[U]^{(j)}=0, \tag{10}$$

where $[M_\perp]$ is the differential-matrix operator, $[R_2]$ is the matrix coefficient

$$[M_\perp]=\begin{pmatrix} -i\omega\rho & \overline{\boldsymbol{\nabla}}_{\cdot}^{(\perp)} & 0 & 0 \\ \overline{\boldsymbol{\nabla}}_s^{(\perp)} & -i\omega\mathbf{S}^E: & -i\omega\mathbf{d}\cdot & 0 \\ 0 & i\omega\mathbf{d}: & i\omega\boldsymbol{\epsilon}^T\cdot & -\boldsymbol{\nabla}_\perp\times \\ 0 & 0 & \boldsymbol{\nabla}_\perp\times & 0 \end{pmatrix},$$

$$[R_2]=\begin{pmatrix} 0 & \overline{\boldsymbol{\nabla}}_{\cdot}^{(\parallel)} & 0 & 0 \\ \overline{\boldsymbol{\nabla}}_s^{(\parallel)} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathbf{e}_z\times \\ 0 & 0 & \mathbf{e}_z\times & 0 \end{pmatrix}, \tag{11}$$

and $[U]$ is the vector function

$$[U]=\begin{pmatrix} \mathbf{v} \\ \mathbf{T} \\ \mathbf{E} \\ \mathbf{H} \end{pmatrix}. \tag{12}$$

Here $\boldsymbol{\nabla}_\perp\times$ is the transverse curl.

We consider Eqs. (4) and (10) as the eigenvalue equations of the main problem and will associate with these equations the eigenvalue equations of the conjugate problem

$$([\tilde{G}_\perp]^{(j)}-\tilde{\gamma}[R_1])[\tilde{V}]^{(j)}=0 \tag{13}$$

and

$$([\tilde{M}_\perp]^{(j)}-\tilde{\gamma}[R_2])[\tilde{U}]^{(j)}=0. \tag{14}$$

Now we define the following scalar products on the waveguide cross sections:

$$\begin{aligned}\int_S([G_\perp][V])\circ[\tilde{V}]^*ds &= \int_S\{(-i\omega\rho\mathbf{v}+\overline{\boldsymbol{\nabla}}^{(\perp)}\cdot\mathbf{T})\cdot\tilde{\mathbf{v}}^*+(\overline{\boldsymbol{\nabla}}_s^{(\perp)}\mathbf{v}-i\omega\mathbf{S}^E:\mathbf{T} \\ &\quad +i\omega\mathbf{d}\cdot\boldsymbol{\nabla}\phi):\tilde{\mathbf{T}}^*-i\omega(\boldsymbol{\nabla}_\perp\cdot\mathbf{D})\tilde{\phi}^*+[-i\omega(\boldsymbol{\epsilon}^T)^{-1}(\mathbf{d}:\mathbf{T}) \\ &\quad +i\omega\boldsymbol{\nabla}_\perp\phi+i\omega(\boldsymbol{\epsilon}^T)^{-1}\mathbf{D}]\cdot\tilde{\mathbf{D}}^*\}ds\end{aligned} \tag{15}$$

and

$$\begin{aligned}\int_S([M_\perp][U])\circ[\tilde{U}]^*ds &= \int_S[(-i\omega\rho\mathbf{v}+\overline{\boldsymbol{\nabla}}^{(\perp)}\cdot\mathbf{T})\cdot\tilde{\mathbf{v}}^* \\ &\quad +(\overline{\boldsymbol{\nabla}}_s^{(\perp)}\mathbf{v}-i\omega\mathbf{S}^E:\mathbf{T}+i\omega\mathbf{d}\cdot\mathbf{E}):\tilde{\mathbf{T}}^* \\ &\quad +(i\omega\mathbf{d}:\mathbf{T}+i\omega\boldsymbol{\epsilon}^T\cdot\mathbf{E}-\boldsymbol{\nabla}_\perp\times\mathbf{H})\cdot\tilde{\mathbf{E}}^* \\ &\quad +(\boldsymbol{\nabla}_\perp\times\mathbf{E})\cdot\tilde{\mathbf{H}}^*]ds.\end{aligned} \tag{16}$$

Based on integration by parts taking into account necessary vector and tensor identities [12], one obtains from Eqs. (15) and (16)

$$\int_{S_j}([G_\perp]^{(j)}[V])\circ[\tilde{V}]^* ds=\int_{S_j}[V]\circ([\tilde{G}_\perp]^{(j)}[\tilde{V}])^* ds + P_1([V],[\tilde{V}]^*) \quad (17)$$

and

$$\int_{S_j}([M_\perp]^{(j)}[U])\circ[\tilde{U}]^* ds=\int_{S_j}[U]\circ([\tilde{M}_\perp]^{(j)}[\tilde{U}])^* ds + P_2([U],[\tilde{U}]^*), \quad (18)$$

where for lossless media

$$[\tilde{G}_\perp]^{(j)}=-[G_\perp]^{(j)}, \quad (19)$$

$$[\tilde{M}_\perp]^{(j)}=-[M_\perp]^{(j)}. \quad (20)$$

P_1 and P_2 in Eqs. (17) and (18) are adjoint bilinear forms. Relations (19) and (20) show that functions $[\tilde{V}]$ and $[\tilde{U}]$ are also (together with functions $[V]$ and $[U]$) included in the domains of definition of operators $[G_\perp]$ and $[M_\perp]$, respectively.

For a regular nonexcited waveguide, one can represent functions $[V],[\tilde{V}]$ and $[U],[\tilde{U}]$

$$[V]=[\hat{V}]e^{-\gamma z}, \quad [\tilde{V}]=[\tilde{\hat{V}}]e^{-\tilde{\gamma}z}, \quad (21)$$

$$[U]=[\hat{U}]e^{-\gamma z}, \quad [\tilde{U}]=[\tilde{\hat{U}}]e^{-\tilde{\gamma}z}, \quad (22)$$

where the functions with carets describe the field distribution over the waveguide cross section (the membrane functions) [12,13].

Let γ_m, $[\hat{V}_m]$, and $[\hat{U}_m]$ be, respectively, the propagation constant and the fields of mode m. We assume also that $\tilde{\gamma}_n$, $[\tilde{\hat{V}}_n]$, and $[\tilde{\hat{U}}_n]$ are, respectively, the propagation constant and the fields of mode n. On the basis of homogeneous electromagnetic and acoustic boundary conditions and conditions at infinity, one obtains zero adjoint bilinear forms in Eqs. (17) and (18). As a result, we have the mode orthogonality relation

$$(\gamma_m+\tilde{\gamma}_n^*)N_{mn}=0. \quad (23)$$

For $\gamma_m+\tilde{\gamma}_n^*\neq 0$, two modes are orthogonal. If, however, $\gamma_m+\tilde{\gamma}_n^*=0$, we will mark $n=\tilde{m}$ and consider this mode as the conjugate to mode m. For conjugate modes, we have an expression for the norm [12]

$$\begin{aligned} N_m\equiv N_{m\tilde{m}}&=\sum_j \int_{S_j}(-\hat{\mathbf{v}}_{\tilde{m}}^*\cdot\hat{\mathbf{T}}_m-\hat{\mathbf{v}}_m\cdot\hat{\mathbf{T}}_{\tilde{m}}^*+i\omega\hat{\mathbf{D}}_m\hat{\phi}_{\tilde{m}}^* \\ &\quad -i\omega\hat{\mathbf{D}}_{\tilde{m}}^*\hat{\phi}_m)\cdot\mathbf{e}_z ds \\ &=\sum_j \int_{S_j}(-\hat{\mathbf{v}}_{\tilde{m}}^*\cdot\hat{\mathbf{T}}_m-\hat{\mathbf{v}}_m\cdot\hat{\mathbf{T}}_{\tilde{m}}^*+\hat{\mathbf{E}}_m\times\hat{\mathbf{H}}_{\tilde{m}}^* \\ &\quad +\hat{\mathbf{E}}_{\tilde{m}}^*\times\hat{\mathbf{H}}_m)\cdot\mathbf{e}_z ds. \end{aligned} \quad (24)$$

We have obtained the mode orthogonality relations based on a rigorous statement of eigenvalue problems in piezoelectric waveguides. Relations (23) and (24) are correct for propagating, evanescent (reactive), and complex modes. We will use these relations to solve the problems of mode excitation by the external electric and magnetic fields.

An analysis of excitation of piezoelectric-waveguide modes by the external electric field is based on the solution of an inhomogeneous equation

$$[G][V]=[Q], \quad (25)$$

where $[G]=[G_\perp]+(\partial/\partial z)[R_1]$ and $[Q]$ is the vector function of sources. In our case of mode excitation by the external fields, $[Q]$ has the form

$$[Q]=\begin{pmatrix} 0 \\ 0 \\ 0 \\ i\omega\mathbf{E}^{\text{exc}} \end{pmatrix}, \quad (26)$$

where $\mathbf{E}^{\text{exc}}$ is the exciting electric field. This field differs from the external electric field $\mathbf{E}^{\text{ext}}$ in Eq. (3) because of the depolarizing effects. Finding the components of the depolarizing tensor represents a separate electrostatic problem.

We will find a solution of Eq. (25) with the use of a complete set of orthonormal membrane eigenfunctions

$$V^{(j)}=\sum_{m=1}^{\infty} a_m(z)\hat{V}_m^{(j)}. \quad (27)$$

Based on this representation, taking into account the orthogonality relations (23) and (24), one obtains the excitation equation for mode m using Galerkin's method [17,18]

$$\frac{da_m(z)}{dz}+\gamma_m a_m(z)=\frac{i\omega}{N_m}\sum_j \int_{S_j}\mathbf{E}^{\text{exc}}\cdot(\hat{\mathbf{D}}_{\tilde{m}}^{(j)})^* ds. \quad (28)$$

An analysis of mode excitation based on the operator $[G]$ and the vector-function $[V]$ cannot be extended to a case of mode excitation by the external magnetic field. For this type of source, we have to use the operator $[M]=[M_\perp]+(\partial/\partial z)[R_2]$ and the vector function $[U]$. One can see from the orthogonality relations (23) and (24) that only transverse components of the electric and magnetic mode fields form a complete set of the orthonormal eigenfunctions. Thus we have

$$\mathbf{E}_\perp^{(j)}=\sum_{m=1}^{\infty} a_m(z)\hat{\mathbf{E}}_{m_\perp}^{(j)}, \qquad \mathbf{H}_\perp^{(j)}=\sum_{m=1}^{\infty} a_m(z)\hat{\mathbf{H}}_{m_\perp}^{(j)}. \quad (29)$$

Since the field $\mathbf{E}$ is the potential field, the coefficients $a_m(z)$ in Eq. (29) are the same as the coefficients in Eq. (27). It becomes clear from the following consideration. Because of Eq. (27), we have

$$\phi^{(j)}=\sum_{m=1}^{\infty} a_m(z)\hat{\phi}_m^{(j)} \quad (30)$$

and therefore

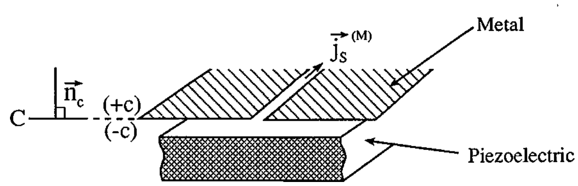

FIG. 2. Piezoelectric waveguide structure.

$$\mathbf{E}_{\perp}^{(j)} = -\sum_{m=1}^{\infty} a_m(z) \nabla_{\perp} \hat{\phi}_m^{(j)} = \sum_{m=1}^{\infty} a_m(z) \hat{\mathbf{E}}_{m_{\perp}}^{(j)}. \quad (31)$$

Let an aperture in a metallic screen of a piezoelectric waveguide take place (Fig. 2). In this case, waveguide eigenmodes are accompanied by surface magnetic currents $\mathbf{j}_s^{(M)}$ [16]. For mode $\tilde{m}$, for example, we can write

$$\mathbf{n}_c \times (\mathbf{E}_{\tilde{m}}^{(+c)} - \mathbf{E}_{\tilde{m}}^{(-c)}) = -\mathbf{j}_{s_{\tilde{m}}}^{(M)}. \quad (32)$$

Here $\mathbf{n}_c$ is the external normal to contour C, the contour on the waveguide cross section. The surface magnetic currents are located on this contour (see Fig. 2). In a piezoelectric waveguide without dissipative losses we can write

$$\mathbf{n}_c \times (\mathbf{H}_{\tilde{m}})_c = 0. \quad (33)$$

We introduce a system of two fields $[U_1]$ and $[U_2]$. Without bulk sources we have

$$[M][U_{1,2}] = \left([M_{\perp}] + \frac{\partial}{\partial z} [R_2] \right) [U_{1,2}] = 0. \quad (34)$$

Based on Eq. (34) we have

$$\int_{S_j} \{ ([M_{\perp}][U_1]) \circ [U_2]^* + [U_1] \circ ([M_{\perp}][U_2])^* \} ds + \int_{S_j} \left\{ \left(\frac{\partial}{\partial z} [R_2][U_1] \right) \circ [U_2]^* + [U_1] \circ \left(\frac{\partial}{\partial z} [R_2][U_2]^* \right) \right\} ds = 0. \quad (35)$$

(Here we have omitted for simplicity the index j for $[M_{\perp}]$ and $[U_{1,2}]$.)

Let the field $[U_1]$ be the required field. In the presence of the external magnetic field, the total magnetic field may be written as

$$\hat{\mathbf{H}}_1 = \sum_{m=1}^{\infty} a_m(z) \hat{\mathbf{H}}_m + \mathbf{H}^{\text{ext}}. \quad (36)$$

For the mode field $\mathbf{H}_m = a_m(z) \hat{\mathbf{H}}_m$, we have the boundary condition $\mathbf{n}_c \times (\mathbf{H}_m)_c = 0$. Let the field $[U_2]$ be the field of mode $\tilde{m}$ that satisfies the boundary conditions (32) and (33). Based on Eq. (34), we have as a result the excitation equation

$$\frac{d a_m(z)}{dz} + \gamma_m a_m(z) = \frac{1}{N_m} \int_C \mathbf{H}^{\text{ext}} \cdot (\mathbf{j}_{s_{\tilde{m}}}^{(M)})^* dc. \quad (37)$$

V. DYADIC POLARIZABILITIES OF PARTICLES

When a piezoelectric resonator with an aperture in a metallic screen is composed by piezoelectric waveguide sections, a problem arises concerning resolution of differential equations (28) and (37) in every waveguide section with sewing together the fields on the boundaries between these sections.

One of the main goals of this paper is to show physically and mathematically that realization of bianisotropic particles based on piezoelectric resonators is possible, in principle. So, we will not describe here any algorithm to calculate the fields in a resonator and will suppose that such calculations have technological capability. When the fields are known, one can find an electric polarization in a piezoelectric and obtain the dyadic polarizabilities α_{ee} and α_{em} and, thus, the induced electric dipole moment p [see (3)] as a result of integration of the electric polarization over the volume of a piezoelectric.

The dyadic polarizabilities α_{me} and α_{mm} [see Eq. (3)] are found on the basis of integration of the surface magnetic charge density $\tau^{(M)}$ along the aperture. The surface magnetic charge density is related to the surface magnetic current density as

$$\nabla s \cdot \mathbf{j}_s^{(M)} = i \omega \tau^{(M)}, \quad (38)$$

where $\nabla_s \cdot$ is a surface divergence on the aperture. The surface magnetic current density is defined as

$$\mathbf{n}_c \times [(\nabla \phi)^{(+)} - (\nabla \phi)^{(-)}] = \mathbf{j}_s^{(M)}, \quad (39)$$

where $(\nabla \phi)^{(\pm)}$ is a gradient of the potential ϕ on the slot region above and below the contour C (see Fig. 2).

Based on piezoelectric resonators with an aperture, bianisotropic materials may be composed of randomly distributed particles or as a three-dimensional regular array of particles. In the latter case, the Lorenz-Lorentz theory may be used to characterize material parameters of artificial bianisotropic crystals [6,19].

VI. CONCLUSION

In this paper we have shown that together with recently conceptualized magnetostatically controlled bianisotropic materials, electrostatically controlled bianisotropic materials can also be introduced. So we have dual curl-free-field bianisotropic composite materials.

Our analysis of dyadic polarizabilities of a piezoelectric bianisotropic particle was mainly based on the theory of excitation of piezoelectric waveguides by the external electric and magnetic fields. It was supposed that when the theory of waveguide excitation is obtained, further calculations of the fields in a resonator have technological capability. To manufacture the ECBMs, well-developed technology of piezoelectric devices (for example, planar technology of surface-acoustic-wave filters [20]) may be successfully used.

The MCBMs and the ECBMs are compositions of microscopic oscillators and the quantum mechanical models can be applicable to describe dynamical perturbations of a system of these oscillations by an external action. Some discussions about the use of the quantum mechanical models for banisotropic particles are adduced in [21]. One can distinguish geometrically symmerical bianisotropic particles [Figs. 1(a) and 1(c)] and bianisotropic particles with symmetry breaking (enantiomers) [Figs. 1(b) and 1(d)]. So the main subject concerns the possibility to use a symmetry analysis to define energetic levels of every bianisotropic particle, similar to an analysis applied for the classification of molecular terms.

[1] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media* (Artech House, Boston, 1994).
[2] A. Lakhtakia, *Beltrami Fields in Chiral Media* (World Scientific, Singapore, 1994).
[3] F. Mariotte, S. A. Tretyakov, and B. Sauviac, Microwave Opt. Technol. Lett. **7**, 861 (1994).
[4] E. O. Kamenetskii, Microwave Opt. Technol. Lett. **11**, 103 (1996).
[5] E. O. Kamenetskii, in Vol. 28 of *NATO Advanced Study Institute, Series 3: High Technology, Advances in Complex Electromagnetic Materials*, edited by A. Priou, A. Sihvola, S. Tretyakov, and V. Vinogradov (Kluwer Academic, Dordrecht, 1997), p. 359.
[6] E. O. Kamenetskii, in *Proceedings of BIANISOTROPICS '97, The International Conference and Workshop on Electromagnetics of Complex Media, Glasgow, 1997*, edited by W. S. Weiglhofer (University of Glasgow, Glasgow, 1997), p. 201.
[7] A. Lakhtakia (private communication).
[8] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).
[9] A. A. Barybin, J. Appl. Phys. **46**, 1707 (1975).
[10] A. Gurevich and G. Melkov, *Magnetic Oscillations and Waves* (Nauka, Moscow, 1994).
[11] D. D. Stancil, *Theory of Magnetostatic Waves* (Springer, New York, 1993).
[12] B. A. Auld, *Acoustic Fields and Waves in Solids* (Wiley, New York, 1973), Vols. 1 and 2.
[13] G. S. Kino, *Acoustic Waves: Devices, Imaging and Analog Signal Processing* (Prentice-Hall, Englewood Cliffs, NJ, 1987).
[14] E. O. Kamenetskii, Microwave Opt. Technol. Lett. **11**, 99 (1996).
[15] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves* (Prentice-Hall, Englewood Cliffs, NJ, 1973).
[16] L. A. Vainstein, *Electromagnetic Waves* (Radio i svyas, Moscow, 1988).
[17] S. G. Mikhlin, *Variational Methods in Mathematical Physics* (Nauka, Moscow, 1970).
[18] C. S. J. Fletcher, *Computational Galerkin Methods* (Springer, New York, 1984).
[19] E. O. Kamenetskii, Phys. Rev. E **57**, 3563 (1998).
[20] *Surface Wave Filters*, edited by H. Mathews (Wiley, New York, 1977).
[21] E. O. Kamenetskii, following paper, Phys. Rev. E **58**, 7965 (1988).